Odds Ratios and Risk Ratios: How are they similar? How are they different?

1 Introduction

Odds ratios and risk ratios are often used when summarizing associations between two variables, one of which is binary. This is a common scenario in epidemiological studies where the goal is to assess the effect of a certain treatment or exposure on the likelihood that a patient has or develops a disease. In this example, disease status is the binary variable of interest and treatment is the covariate which, in this case, is also binary. The distinction between risk ratios and odds ratios can be confusing and researchers are often not sure which one is the appropriate statistic for their study. In this newsletter we will review the definitions of risk ratios and odds ratios, compare the two statistics, and discuss the situations in which each is appropriate.

2 Risk ratio

Suppose we are interested in assessing the effectiveness of a new vaccine for influenza. We identify 48 individuals that have received the vaccine and 40 that haven't and then follow the subjects to see which become infected. The table below summarizes our findings.

Table 2.1: Number of infected and not infected invidiaula by vaccination status.

	Infected	Not infected
Vaccinated	$a = 16$	$b = 32$
Not Vaccinated	$c = 30$	$d = 10$

The risk (or probability) of becoming infected can be estimated as the number of subjects that become infected divided by the total number of subjects observed. Based on the table above, the estimated risk of becoming infected for the vaccinated group is $a/(a+b) = 16/(16+32) = 0.33$ and the risk of becoming infected for the unvaccinated group is $c/(c+d) = 30/(30+10) = 0.75$. The risk ratio (or relative risk) of becoming infected for the vaccinated group versus the unvaccinated group is the ratio of these two values, which is

$$\text{RR} = \frac{\text{risk in vaccinated group}}{\text{risk in unvaccinated group}} = \frac{a/(a+b)}{c/(c+d)} = \frac{0.33}{0.75} = 0.44$$

The interpretation of the RR is that individuals who are vaccinated are 0.44 times as likely to become infected compared to individuals that have not been vaccinated. Since the RR is less than 1, the vaccination is protective against influenza. An RR that is greater than 1 would indicate that vaccination makes infection more likely and an RR equal to 1 indicates that the vaccination has no effect on the risk of infection. Notably, the RR must be non-negative and is bounded above by the inverse risk in the unvaccinated group, in this case $1/0.75 = 1.33$.

3 Odds ratio

Alternatively, the association between vaccination and risk of infection can be summarized using an odds ratio. The odds of becoming infected can be estimated as the probability of being infected divided by the probability of not being infected. Using the summary table above, the odds of a vaccinated subject becoming infected are $\frac{a/(a+b)}{b/(a+b)} = a/b = 16/32 = 0.5$ and the odds of an unvaccinated subjected becoming infected are $\frac{c/(c+d)}{d/(c+d)} = c/d = 30/10 = 3.0$. The odds ratio comparing a vaccinated subject to an unvaccinated subject is given by

$$\text{OR} = \frac{\text{odds of infection in vaccinated group}}{\text{odds of infection in unvaccinated group}} = \frac{a/b}{c/d} = \frac{ad}{bc} = \frac{0.5}{3.0} = 0.17.$$

The OR tells us that the odds of a vaccinated subject becoming infected are 0.17 times the odds for an unvaccinated subject. As with the RR, an OR less than 1 indicates that the vaccine is protective against the infection, an OR greater than 1 indicates that the vaccine increases risk of infection, and an OR of 1 indicates that there is no association between vaccination and infection. The OR must be non-negative but, unlike the RR, the OR has no upper bound.

As can be seen in the example, the RR and the OR are not the same. In fact, using a bit of algebra one can see that

$$\text{OR} = \text{RR} \times \left(\frac{1 - a/(a+b)}{1 - c/(c+d)}\right).$$

Thus the OR and RR are only the same when $a/(a+b) = c/(c+d)$; that is when the risk of infection is the same for both groups. However, if both *a* and *c* are very small, which will occur when the number of cases of infection is small for both groups, then the OR and the RR will be similar (a good rule of thumb is when the prevalence of the disease is less than 10 percent). Notably, if the risk of disease is smaller in the treated group, as is often the case in studies attempting to establish the effectiveness of a treatment, the OR will be smaller than the RR.

4 Cohort versus case-control studies

The study described above is an example of a cohort study, a study in which members of a population of interest with certain characteristics (here, vaccinated and unvaccinated) are identified and then the response variable is obtained (here, infected or not infected). Risk ratios and odds ratios are appropriate for cohort studies as well as for randomized trials and cross-sectional studies. For all three of these study designs, the RR and the OR can be used because the subjects were not selected for study participation based on their disease status and hence the risk of disease can be estimated.

Suppose that we had decided to use a case-control study instead. A case-control study would involve selecting a sample of individuals that had been infected and a sample of subjects that were not infected. Then information on their prior vaccination status would be obtained. Since the participants were selected based on infection status, we cannot calculate the risk of infection in the vaccinated and unvaccinated groups. Instead we can calculate the odds ratio for vaccination to assess the association between vaccination and infection.

The odds that an infected subject was vaccinated are $a/c = 16/30 = 0.53$ and the odds that an uninfected subject was vaccinated are $b/d = 32/10 = 3.2$. The vaccination odds ratio for the infected group versus the uninfected group is

$$\text{OR} = \frac{\text{odds of vaccination in infected group}}{\text{odds of vaccination in uninfected group}} = \frac{a/c}{b/d} = \frac{ad}{bc} = \frac{0.53}{3.2} = 0.17.$$

Notably, the formula for this OR is identical to the OR for infection, although the interpretation is different due to the way in which the subjects were sampled for the case-control study.

In summary, risk ratios and odds ratios can be used when study participants are not selected based on the outcome of interest (e.g., infection status). However, odds ratios are the only measurement of association that can be calculated when participants have been selected based on the outcome of interest (such as in case-control studies) and hence the sampling does not allow us to estimate the true proportion of diseased individuals within the population.

Estimates of OR and RR while controlling for other covariates can be obtained using logistic regression and other generalized linear models.

Selected References:

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Created December 2015. Last updated April 2022.