

MixedModR2

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Generate the Data

Generate 1000 data points from a population with one random effect:

20 levels of Factor A, each sampled 50 times

```
set.seed(39)

siga <- 50
sige <- 30
beta0 <- 100
beta1 <- 3
beta2 <- 1.2
Alevel <- rep(paste("A", 1:20, sep=""), each=50)
effA <- rep(rnorm(20, 0, siga), each=50) #level a

x1 <- rnorm(1000, 0, 8) #obs level
eps <- rnorm(1000, 0, sige)

y <- beta0 + beta1*x1 + effA + eps
```

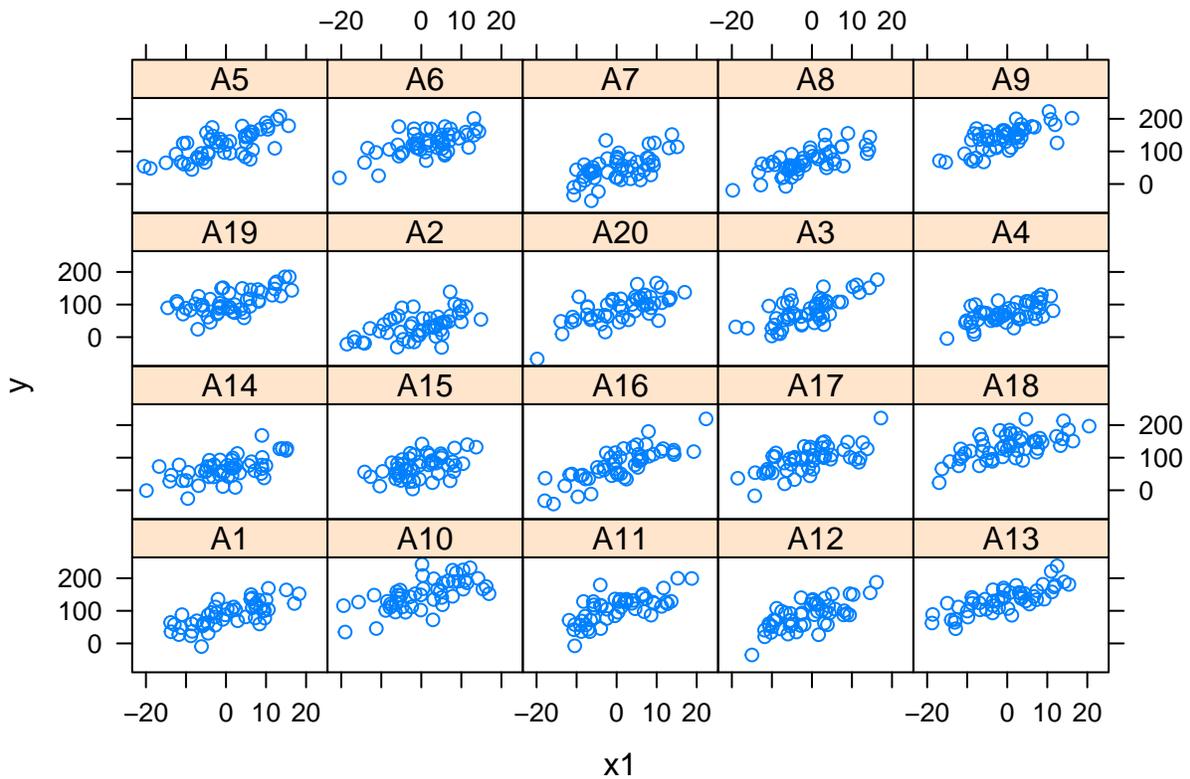
This matches the equation

$$\begin{aligned} y_{ij} &= \beta_0 + \beta_1 x_{1ij} + \alpha_j + \varepsilon_{ij} \\ &\alpha_j + \text{Gaussian}(0, \sigma_\alpha^2) \\ &\varepsilon_{ij} + \text{Gaussian}(0, \sigma_\varepsilon^2) \end{aligned}$$

Where $\beta_0 = \text{beta0}$

```
dat <- data.frame(Alevel, effA, x1, y)
write.csv(dat, file="R2trial.csv")

library(lattice)
xyplot(y~x1|Alevel, data=dat)
```



Fit the models

Fit a model with x1 as the fixed effect (mod1) and a null model (mod0)

Model 1

$$\begin{aligned}
 y_{ij} &= \beta_0 + \beta_1 x_{1ij} + \alpha_j + \varepsilon_{ij} \\
 \alpha_j &+ \text{Gaussian}(0, \sigma_\alpha^2) \\
 \varepsilon_{ij} &+ \text{Gaussian}(0, \sigma_\varepsilon^2)
 \end{aligned}$$

```

library(lme4)

mod1 <- lmer(y ~ x1 + (1|Alevel), data=dat)
summary(mod1)

```

```

Linear mixed model fit by REML ['lmerMod']
Formula: y ~ x1 + (1 | Alevel)
Data: dat

```

REML criterion at convergence: 9682.5

```

Scaled residuals:
  Min       1Q   Median       3Q      Max

```

-2.9129 -0.6551 0.0473 0.6248 3.1789

Random effects:

Groups	Name	Variance	Std.Dev.
Alevel	(Intercept)	956.5	30.93
Residual		870.8	29.51

Number of obs: 1000, groups: Alevel, 20

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	95.1638	6.9784	13.64
x1	3.3648	0.1181	28.48

Correlation of Fixed Effects:

(Intr)
x1 0.001

Note that the Fixed effect estimates were recovered, more or less. The intercept was estimated to be 95.164 which is close to 100, and the coefficient for X1 was spot-on at 3.

Note I'm still trying to figure out how to recover sigma and sige from this output...

Null Model

$$y_{ij} = \beta_0 + \alpha_j + \varepsilon_{ij}$$
$$\alpha_j + \text{Gaussian}(0, \sigma_\alpha^2)$$
$$\varepsilon_{ij} + \text{Gaussian}(0, \sigma_\varepsilon^2)$$

```
mod0 <- lmer(y ~ 1 + (1|Alevel), data=dat)
summary(mod0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: y ~ 1 + (1 | Alevel)
Data: dat
```

REML criterion at convergence: 10271.7

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.8979	-0.6763	0.0336	0.6533	3.5152

Random effects:

Groups	Name	Variance	Std.Dev.
Alevel	(Intercept)	959.1	30.97
Residual		1590.4	39.88

Number of obs: 1000, groups: Alevel, 20

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	94.899	7.039	13.48

Options for R^2

Obtain the variances at each level

Most of the R^2 equations rely on the variance at the level of observation (residual) and at the level of the A group (effA). Extract the variance tables from each of the models, and save the pieces we want (under the vcov) heading in easier to handle names:

```
(vars0 <- as.data.frame(VarCorr(mod0)))

      grp      var1 var2      vcov      sdcov
1  Alevel (Intercept) <NA> 959.1122 30.96954
2 Residual           <NA> <NA> 1590.4469 39.88041

vara_0 <- vars0[1,4]
vare_0 <- vars0[2,4]
```

Here `vara_0` is the variance for level a for the null model $\sigma_{a0}^2 = 959.112$ and `vare_0` is the variance for the residual $\sigma_{\varepsilon_0}^2 = 1590.447$

Similarly we get the same information for the model with the fixed effect:

```
(vars1 <- as.data.frame(VarCorr(mod1)))

      grp      var1 var2      vcov      sdcov
1  Alevel (Intercept) <NA> 956.5491 30.92813
2 Residual           <NA> <NA> 870.7986 29.50930

vara_1 <- vars1[1,4]
vare_1 <- vars1[2,4]
```

Here `vara_1` is the variance for level a for the null model $\sigma_a^2 = 956.549$ and `vare_1` is the variance for the residual $\sigma_\varepsilon^2 = 870.7986$

Now we explore several R^2 options outlined in Nakagawa & Schielzeth 2013

These equations were proposed by Snijders & Bosker (1994) for Linear mixed models with 1 random factor.

Nakagawa and Schielzeth 2013

Nakagawa and Schielzeth 2013 suggest new versions for R^2 . They both rely on σ_f^2 , the variance explained by the fixed effect components. This is estimated by the variance of the predictions for the data without worrying about the random effects. This can also be estimated by multiplying the design matrix for the fixed effects by the vector of fixed effects estimates.

$$\sigma_f^2 = \text{var}(\beta_1 x_1)$$

```
varf_1 <- var(predict(mod1, re.form=NA))

#alternate method
#varf_1 <- var(as.vector(lme4::fixef(mod1) %*% t(mod1@pp$X)))
```

Marginal R^2 (whole model)

$R^2_{LMM(m)}$ is the marginal R^2 for a linear mixed model, meaning that it is concerned with the variance explained by the fixed factors:

$$R^2_{LMM(m)} = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\alpha^2 + \sigma_\varepsilon^2}$$

```
(R2LMMm <- varf_1/(varf_1+vara_1+vare_1))
```

```
[1] 0.281427
```

Conditional R^2 (whole model)

$R^2_{LMM(c)}$ is the conditional R^2 for a linear mixed model, meaning that it is concerned with the variance explained by the fixed *and random* factors:

$$R^2_{LMM(c)} = \frac{\sigma_f^2 + \sigma_\alpha^2}{\sigma_f^2 + \sigma_\alpha^2 + \sigma_\varepsilon^2}$$

```
(R2LMMc <- (varf_1+vara_1)/(varf_1+vara_1+vare_1))
```

```
[1] 0.6575735
```

We can also calculate these directly with the MuMIn package.

```
library(MuMIn)
```

```
r.squaredGLMM(mod1)
```

```
      R2m      R2c  
[1,] 0.281427 0.6575735
```

Proportion change in variance at each level of variance

The authors also advocate for looking at the proportion change in variance at each level, which looks at the ratio of variances at each level for the model with fixed-effects to the variances at each level for the null model.

$$PCV_\alpha = 1 - \frac{\sigma_\alpha^2}{\sigma_{\alpha 0}^2}$$
$$PCV_\varepsilon = 1 - \frac{\sigma_\varepsilon^2}{\sigma_{\varepsilon 0}^2}$$

```
(PCVa <- 1 - vara_1/vara_0)
```

```
[1] 0.002672369
```

```
(PCVe <- 1 - vare_1/vare_0)
```

```
[1] 0.4524819
```